Intangible Assets, Knowledge Spillover, and Markup

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Abstract

Intangible assets (e.g. software) have unique characteristics compared to physical capital; they are scalable and exhibit spillover effects. This paper develops a structural model to empirically test these features of intangible assets. I introduce intangible capital into the production function as an additional factor input and external knowledge as a productivity shifter. I estimate production functions at the firm level including labor-augmenting, and Hicks-neutral productivity without imposing any parametric functional form. My empirical results indicate a positive and significant impact of intangible capital on a firm's production. This return to intangibles increases with firm size in all sectors, suggesting that intangible capital exhibits scalability. Moreover, knowledge spillovers increase firm productivity, and the extent of this increase varies depending on firm size, and sector. Large firms and firms in the health sector tend to benefit more from their rival's knowledge stock. Additionally, I reveal that markups rise with a firm's intangible intensity, suggesting a potential explanation for the recent rise in market concentration.

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1 Introduction

Intangible capital has become an important part of firms' capital stock over the last few decades (Corrado et al. (2005), Hulten and Hao (2008)). Intangible assets encompass a diverse range of components, including, but not limited to, patents, software, databases, product design, firm-specific human capital, organizational structure of the company, and distribution systems. It is named 'capital' because firms invest in them to produce more output today and in the future. They are 'intangible' due to their lack of physical presence, a characteristic shared by all the examples above.

What sets intangible capital apart from physical capital is its *non-rivalry in use* feature due to its lack of physical presence. Intangible capital can be interpreted as pieces of information, and firms require a storage medium to use them in their production (Crouzet et al. (2022)). The medium can be in the form of capital (a computer to use software), a document (for a patent or a design), or a person (for an innovation). This unique feature allows firms to use the same intangible capital in multiple production processes simultaneously. To illustrate this, consider a company where the product's design is transmitted to various machines for production. These machines simultaneously execute the commands received through software. The software, representing intangible capital, is employed in different production processes at the same time. In contrast, each machine, a form of physical capital, can only participate in the production process one at a time. This non-rival characteristic of intangible assets not only sets them apart but also empowers firms with scalability and enables economies of scale.

While intangible assets are utilized simultaneously across different production processes within a firm, this doesn't prevent other firms from imitating the same intangible capital in their own businesses by copying software or acquiring information, for example. This gives rise to a second characteristic of intangibles, known as *limited excludability* (Crouzet et al. (2022)). Patents and copyrights provide a property right for firms to protect their ideas and creations. However, it's important to recognize that even with a patent in place, the benefits of these intangibles can often extend beyond the patent holder. Other firms can still benefit indirectly from the patented idea in various ways. They may explore the patented technology for insights, develop their own innovations, or build complementary products or services. The limited excludability feature, therefore, generates a spillover effect across firms.

Intangible capital consists mainly of two parts: knowledge capital and organizational capital (Peters and Taylor (2017a)). The knowledge capital is the intangible value of a company, comprising its knowledge, learned techniques, procedures, and innovations. Firms invest in research and development (R&D) to expand their knowledge stock, aiming to generate product or process innovation.

The organizational capital, on the other hand, comprises the intangible assets within a company, including its management practices, workforce expertise, culture, internal systems, and external relationships. These elements collectively contribute to the organization's operational and managerial efficiency. Some examples of organizational capital include IBM's extensive system of selling or licensing know-how, Zara's process of transmitting real time customers' choices to its suppliers worldwide, and Amazon's highly efficient distribution systems. A common thread among these business processes and practices is that they are not easily mimicked by competitors (Lev et al. (2016)). The organizational capital cannot be completely codified and hence transferred to other organizations or imitated by them. (Lev and Radhakrishnan (2003)). It represents a factor of production that is unique to the firm.

The distinction between knowledge and organizational capital plays a crucial role in understanding the limited-excludability feature of intangible assets. Since organizational capital is inherently firm-specific and challenging to transfer, it lacks the limited-excludability feature of intangibles. In contrast, knowledge capital, which can be acquired or imitated by other firms, becomes the primary factor through which limited excludability can be realized (Bloom et al. (2013)). Therefore, the spillovers among firms will predominantly occur through the knowledge capital, not the organizational capital.

In this paper, I develop a model to estimate the impact of intangible capital on a firm's output production, taking into account its knowledge spillover effects among firms. Recognizing that intangibles possess distinct attributes compared to physical capital, I introduce them as an additional factor input in production, rather than defining capital as the sum of intangible and physical assets. This approach allows me to estimate output elasticity of intangible capital and examine whether this effect varies with firm size, thus providing an assessment of the scalability of intangibles.

The change in a firm's intangible capital corresponds to a movement along the production curve, similar to the other factor inputs. However, the knowledge spillover from other firms shifts the entire production curve because they are external shocks to the firm's optimal production decisions. I, therefore, introduce knowledge spillover into the production function as a variable that shifts the firm's productivity. The shift in productivity through knowledge spillovers can exhibit bias toward some factor inputs. The labor productivity tends to be more affected by the influence of knowledge spillovers than the other factor inputs. Doraszelski and Jaumandreu (2018) provide evidence that technological improvements are biased toward the labor productivity. To address these dynamics, I make a clear distinction between two types of productivity: Hicks-neutral and labor-augmenting productivity. My model accommodates these productivity biases, recognizing that knowledge spillovers may exert a more substantial influence on the labor productivity relative to the other factor inputs.

I estimate nonparametric production functions that incorporate labor-augmenting and Hicks-neutral productivity following Demirer (2020) methodology. Rather than relying on strict parametric assumptions for the production function, I impose a functional form assumption that encompasses the typical parametric models. This allows me to estimate the intangible elasticity of output for each individual firm, facilitating to test the scalability of intangibles. I offer two novel contributions on top of Demirer (2020). Firstly, I introduce intangible capital as an additional factor input within the production function. Second, I account for the impact of other firms' knowledge capital on a firm's productivity. Importantly, this implies that productivity is not solely determined by an exogenous Markov process but is also influenced by external knowledge capital.

My estimation results indicate that intangible capital significantly increases the firm's output, with the effect becoming more pronounced as the firm's size increases. This reflects the scalability of intangibles. I also conduct an estimation of the effect of knowledge spillover on firm productivity and find that firms are positively and significantly affected by the knowledge capital stock of other firms. This influence depends on both firm size and the intangible intensity of the firms. Larger firms tend to benefit more from the existing knowledge stock of other firms, and similarly, intangible-intensive firms are also able to derive greater benefits from the existing knowledge stock in the economy.

This paper contributes to the markup estimation literature using production function estimation (Hall (1988), Loecker and Warzynski (2012), De Loecker et al. (2020), Raval (2022)). This literature defines the markup as a ratio of flexible input elasticity to the flexible input share in revenue using cost-minimization problem. Since the introduction of intangible capital into the production function affects output elasticities, it also affects the markup rates that firms charge. I demonstrate that firms with higher intangible intensity tend to charge higher markup rates, which aligns with the central role of intangible assets. Intangibles like organizational or knowledge capital significantly reduce firms' marginal costs (De Ridder (2019)). Consequently, firms can charge higher markups as the markup is price over marginal cost. The recent surge in markup rates appears to be primarily driven by intangible-intensive firms and sectors, such as high-tech and healthcare. Furthermore, I estimate the weighted average of markups and find lower aggregate markups compared to the literature.

I contribute to the firm level intangible capital literature (Haskel and Westlake (2018), Lev and Radhakrishnan (2005), Corrado et al. (2009), Crouzet et al. (2022)). While existing literature assumes that intangible capital is scalable and exerts spillover effects on other firms, these assumptions haven't been rigorously tested using empirical data. I complement this literature by providing an estimation method and by empirically estimating these features using firm-level data.

This paper contributes to the knowledge spillover literature (Bernstein and Nadiri (1988), Bloom et al. (2013), Henderson et al. (2005)). The conventional approach has been to create an external knowledge stock metric and integrate it as an additional factor input into the production function. Alternatively, they have utilized patent data to estimate its impact on firm productivity. I contribute to the literature introducing the external knowledge capital into the production function as a productivity shifter, inspired by endogenous growth literature Romer (1990), rather than another factor input.

This paper is organized as follow: Section 2 outlines the model and assumptions required to estimate the model. Section 3 explains how I identify output elasticities, markup and the effect of knowledge spillover on the firm's productivity. Section 4 introduces the data and estimation strategy. Section 5 presents the results of production function estimation. Section 6 concludes.

2 Model

This section presents the production function employed by firms. I follow Demirer (2020) for the model and estimation setup. I extend his paper adding intangible capital into the production function and controlling for knowledge spillovers across firms.

2.1 Production Function

Firm i produces output at year t with the following production function

$$Y_{it} = F_t(K_{it}, K_{it}^{int}, \omega_{it}^L L_{it}, M_{it}) exp(\omega_{it}^H) exp(\epsilon_{it}) \tag{2.1}$$

where Y_{it} is firm's output, K_{it} is physical capital , K_{it}^{int} is intangible capital, M_{it} is material, L_{it} is labor, and $\omega_{it}^L \in \mathbb{R}$ is labor-augmenting productivity. $\omega_{it}^L L_{it}$ can be interpreted as effective labor. Labor productivity is not the only productivity in the production process. $\omega_{it}^H \in \mathbb{R}$ is Hicks-neutral productivity, increasing firm output for any given set of factor inputs. $\epsilon_{it} \in \mathbb{R}$ represents an exogenous random shock to the production.

The model encompasses two types of production inputs: predetermined and flexible inputs. Physical and intangible capital are classified as *predetermined* variables. Firms decide their levels prior to the beginning of the year. This decision affects future production. In contrast, material and labor are assumed *flexible* inputs. Firms take their flexible input decision each year in order to minimize their production cost given their information set, denoted by $\mathcal{I}_{it}$. The information set includes labor augmenting productivity, Hicks-neutral productivity, past physical and intangible capital, and past information sets. I assume that the information set is orthogonal to the random shock, i.e. $E[\epsilon_{it} \mid \mathcal{I}_{it}] = 0$. Therefore, the random shock represents a measurement error different from the productivity shocks and is not observed by the firm.

The input prices are determined in a perfectly competitive market, yielding a constant input prices across firms given a year. However, I allow for imperfect competition in the output market, leading to a market power. The firms in the output markets are able to charge markups over their marginal costs.

A distinctive feature of the model is to incorporate intangible capital into the production function. The inclusion of intangible capital as another factor input is crucial, as it allows for a representation of the economies' dependence on knowledge, innovation, and non-physical assets. Intangible capital, encompassing elements such as intellectual property and organizational knowledge, plays an important role in the production function by capturing the increasingly significant contributions of non-physical assets to the firm's output (Corrado et al. (2005)). This role of intangible assets becomes even more pronounced with technological advancements, particularly in recent decades.

The exposure to intangible capital reveals sectoral differences and possesses a time-varying effect on the firm's production (Crouzet and Eberly (2019)). These variations over time and across industries are not unique to intangible capital; instead, other factor inputs also exhibit similar patterns. As an example, Autor et al. (2020) highlights a decrease in the labor share within production. I, therefore, allow for production functions to be *industry-specific* and *time-varying* meaning that firms within a sector share a common production function, which, however, evolves dynamically over time. Industry and time are not the only sources of heterogeneity in production, a substantial *cross-firm* heterogeneity also exists. For instance, large firms tend to be more capital-intense whereas small firms exhibit a greater labor intensity (Holmes and Schmitz (2010)). Non-parametric production function enables me to control for the cross-firm heterogeneity by incorporating factor inputs.

Technological change, with its inherent potential to uniformly enhance the productivity of all factors of production, can also introduce biases favoring specific factor inputs. To navigate this multi-dimensional productivity, I choose to introduce labor-augmenting productivity into the production function along with Hicks-neutral productivity. My empirical strategy accommodates only one flexible factor input productivity. This decision is in line with the seminal paper of Doraszelski and Jaumandreu (2018), which reveals that technological change is biased toward labor-augmenting productivity. Second, the decision is supported by the empirical findings that, labor costs exhibit the most variation among factor inputs across firms, implying a substantial unobserved heterogeneity within labor input. Third, an integral part of intangible capital is knowledge capital, characterized by its limited excludability. This characteristic allows workers across different companies to benefit from the research conducted by their counterparts. As a result, the spillover effect of intangible assets aligns most closely with labor productivity.

2.2 Assumptions

This section discusses assumptions of the model to estimate the production function of the firms. The primary and foundational assumption is the homothetic separability assumption. This permits to define the labor productivity as a function of observed variables. Additionally, I follow the standard assumption of the industrial organization literature and follow Levinsohn and Petrin (2003) methodology (LP) to derive the Hicks-neutral productivity as a function of factors of production. Lastly, I explicitly define the productivity processes and formulate the spillover effect of intangible assets.

Assumption 2.1: Weak Homothetic Separability

I assume that the production function has a functional form of:

$$Y_{it} = F_t(K_{it}, K_{it}^{int}, h_t(\omega_{it}^L L_{it}, M_{it}))exp(\omega_{it}^H)exp(\epsilon_{it}) \tag{2.2}$$

where $h_t(\omega_{it}^L L_{it}, M_{it})$ is a homogeneous function. This assumption states that the production function is separable into two parts. The physical and intangible capital part is separable from a homogeneous function of material and effective labor. This might look restrictive but many of the production functions used in the literature satisfy this assumption. For instance, the Cobb-Douglas production function has

$$Y_{it} = K_{it}^{\alpha}(K_{it}^{int})^{\beta}(\omega_{it}^L L_{it})^{\gamma} M_{it}^{\theta} exp(\omega_{it}^H)exp(\epsilon_{it})$$

$h(.,.)$ function, in this case, is $(\omega_{it}^L L_{it})^\gamma M_{it}^\theta$ and it is homogeneous and separable from $K_{it}^\alpha (K_{it}^{int})^\beta$. It's important to note that the Cobb-Douglas has limitations, including a constant output elasticity of factor inputs across firms and fixed elasticity of substitution at one.

Another commonly used functional form is the Constant Elasticity of Substitution (CES) production function:

$$Y_{it} = \left(\beta_k K_{it}^\sigma + \beta_I (K_{it}^{int})^\sigma + \beta_l (\omega_{it}^L L_{it})^\sigma + (1 - \beta_k - \beta_I - \beta_l) M_{it}^\sigma\right)^{\upsilon/\sigma} exp(\omega_{it}^H) exp(\epsilon_{it})$$

For CES, $h(.,.) = \beta_l (\omega_{it}^L L_{it})^\sigma + (1 - \beta_k - \beta_I - \beta_l) M_{it}^\sigma$ is homogeneous and separable from $\beta_k K_{it}^\sigma + \beta_I (K_{it}^{int})^\sigma$. The CES generates a constant elasticity of substitution σ. While the CES offers greater flexibility compared to the Cobb-Douglas, it still maintains a constant elasticity of substitution and does not yield a heterogeneous output elasticity of factor inputs across firms.

Recognizing the limitations of CES and Cobb-Douglas production function, the acknowledgment of homothetic separability assumption introduces a more general and flexible functional form. This assumption establishes a more extensive basis for modeling heterogeneous production functions.

Assumption 2.2: Firms minimize their costs

Firms decide their optimal material and labor demands minimizing their costs. As physical and intangible capital are predetermined variables they are not included in the cost minimization problem. This transforms the problem into a static one. Another advantage of the cost minimization is that it facilitates to determine the markup levels of firms. As demonstrated by Loecker and Warzynski (2012), markups are derived from the first-order conditions (FOC) of the cost minimization problem.

The firms minimize their cost given input prices and output demand

$$\min_{L_{it}, M_{it}} \left\{ p_t^L L_{it} + p_t^M M_{it} : F_t(K_{it}, K_{it}^{int}, h_t(\omega_{it}^L L_{it}, M_{it})) exp(\omega_{it}^H) exp(\epsilon_{it}) \geq \tilde{Y}_{it} \right\}$$

where p_t^L is the wage rate of the workers at time t and p_t^M is the price of material at time t. As input market is perfectly competitive, the flexible input prices are constant across firms but varying over time. $\tilde{Y}_{it}$ is the output demand from firm i at time t.

Applying Shepherd's lemma and homotheticity assumption, the cost minimization problem induces

$$\tilde{M}_{it} = \frac{M_{it}}{L_{it}} = r_t(\omega_{it}^L) \tag{2.3}$$

where $r_t(.)$ is an unknown function. The proof is in the appendix. The equation posits that the optimal material-to-labor ratio, $\tilde{M}_{it}$, is solely determined by labor-augmenting productivity. It is entirely independent of Hicks-neutral productivity. If the $r_t(.)$ function is invertible, labor-augmenting productivity, ω_{it}^L, can be expressed as a function of flexible input ratio, $\tilde{M}_{it}$. To achieve this inversion of $r_t()$, I require strict monotonicity of the flexible input ratio in labor-augmenting productivity. I now introduce the third assumption needed to define the labor productivity as a function of flexible inputs.

Assumption 2.3: Elasticity of substitution between effective labor and material is less than 1 or greater than 1.

The elasticity of $\tilde{M}_{it}$ with respect to ω_{it}^L is equal to $-\sigma_t(\omega_{it}^L L_{it}, M_{it}) + 1$, where $\sigma_t(\omega_{it}^L L_{it}, M_{it})$ is the elasticity of substitution between effective labor and material. The proof is in the appendix. When $\sigma_t(\omega_{it}^L L_{it}, M_{it}) > 1$ or $\sigma_t(\omega_{it}^L L_{it}, M_{it}) < 1$, $\tilde{M}_{it}$ is always increasing or decreasing function of ω_{it}^L, implying that $\tilde{M}_{it}$ is strictly monotone in ω_{it}^L. Thanks to this assumption, I can express ω_{it}^L as a function of $\tilde{M}_{it}$, represented by

$$\omega_{it}^L = r_t^{-1}(\tilde{M}_{it}) \tag{2.4}$$

Unobserved labor productivity can now be characterized as an unknown function of the material-to-input ratio. Another unobserved variable in the production function is the Hicks-neutral productivity. In order to characterize it as a function of factor inputs I introduce the following monotonicity assumption.

Assumption 2.4: Material demand is monotone in Hicks-neutral productivity

I assume following Levinsohn and Petrin (2003) that

$$M_{it} = s_t(K_{it}, K_{it}^{int}, \omega_{it}^L, \omega_{it}^H) \tag{2.5}$$

where $s_t(.)$ is a strictly increasing function in ω_{it}^H. This states that a firm's material demand increases with its productivity. In other words, more productive firms demand higher materials. This framework introduces two notable innovations, extending the framework of Levinsohn and Petrin (2003). Firstly, I incorporate the labor-augmenting productivity akin to Demirer (2020), thereby accounting for the influence of labor productivity on the marginal product of materials. Second, I introduce intangible capital in conjunction with physical capital as it augments the state-space.

Since material is strictly increasing in Hicks-neutral productivity, I can invert productivity as follows:

$$\omega_{it}^{H} = s_t^{-1}(K_{it}, K_{it}^{int}, \omega_{it}^{L}, M_{it})$$

The Hicks-neutral productivity is expressed as a function of factor inputs, excluding ω_{it}^{L}. From equation 2.4, the labor augmenting productivity is only a function of material to input ratio. Substituting this, the equation becomes

$$\omega_{it}^{H} = s_t^{-1}(K_{it}, K_{it}^{int}, r_t^{-1}(\tilde{M}_{it}), M_{it}) = \bar{s}_t(K_{it}, K_{it}^{int}, \tilde{M}_{it}, M_{it}) \tag{2.6}$$

where $\bar{s}_t(.)$ is an unknown function. ω_{it}^{H} becomes a function of factor inputs, only determined by physical capital, intangible capital, material-to-labor ratio and materials. As a result, it is observable by the econometrician. The next two assumptions are used in my estimation part. I will begin by explaining the model's timing assumption and then introduce productivity shocks.

Assumption 2.5: Physical and intangible capital are predetermined variables

The levels of physical and intangible capital are determined in the preceding year through the following equations:

$$K_{it} = \kappa_{1t}(K_{it-1}, I_{it-1})$$
$$K_{it}^{int} = \kappa_{2t}(K_{it-1}^{int}, I_{it-1}^{int})$$

where I_{it} represents the physical investment and I_{it}^{int} intangible investment of firm i within year t. $\kappa_{1t}(.)$ and $\kappa_{2t}(.)$ are unknown functions. This allows for adjustment costs and fixed costs of investment. This timing assumption implies that firms start to fully realize the benefits of physical and intangible investments within a year in their production processes. This has a time-to-build investment model perspective (Kydland and Prescott (1982)).

Assumption 2.6: Productivity shocks have a first-order Markov process

The productivity shocks follow a first-order Markov process:

$$P(\omega_{it}^L, \omega_{it}^H \,\Big|\, \mathcal{I}_{t-1}) = P(\omega_{it}^L, \omega_{it}^H \,\Big|\, \omega_{it-1}^L, \omega_{it-1}^H, \sum_{j \in I, j \neq i} R_{jt-1})$$

where R_{jt} is the knowledge capital stock of company j at time t. The joint probability of productivity has a first-order Markov process.This framework accounts for the knowledge spillover among firms, where the productivity of company i can be influenced by the knowledge capital stock of other companies. Thus, the Markov process is contingent not only on the firm's own history but also on the cumulative knowledge capital stocks of other firms. Intuitively, when a company develops a new technology other firms can benefit from this knowledge (Bloom et al. (2013)). This increases the other firms' probability of having higher productivity that operates in similar industries. (Bernstein and Nadiri (1988)) This is driven by the inherent nature of knowledge capital which has limited excludability. Firms can only partially exclude others from utilizing their knowledge.

I assume productivity shocks have a continuous function. Using the Skorohod representation of random variables, the labor productivity process can be defined as

$$\omega_{it}^L = g_1(\omega_{it-1}^L, \omega_{it}^H, \sum_{j \in I, j \neq i} R_{jt-1}, u_{it}^1), \quad u_{it}^1 \mid \omega_{it-1}^L, \omega_{it}^H, \sum_{j \in I, j \neq i} R_{jt-1} \sim \text{Uniform}(0,1)$$

u_{it}^1 can be seen as an innovation term for productivity. Compared to the standard productivity assumption, this shock is not separable from past productivity and spillover effect. It has a uniform distribution conditioning on past productivity and spillover effects. This allows me to define the control variables in the empirical part. Alternatively, u_{it}^1 can be interpreted as a firm's productivity rank after controlling for its past productivity, and spillover effect through knowledge capital.

Similarly, I can define Hicks-neutral productivity as

$$\omega_{it}^H = g_2(\omega_{it-1}^L, \omega_{it-1}^H, \sum_{j \in I, j \neq i} R_{jt-1}, u_{it}^1, u_{it}^2), \quad u_{it}^2 \mid \omega_{it-1}^L, \omega_{it}^H, \sum_{j \in I, j \neq i} R_{jt-1}, u_{it}^1 \sim \text{Uniform}(0,1)$$

u_{it}^2 shares similar characteristics with u_{it}^1. By the Skorohod representation, u_{it}^2 is uniformly distributed conditioning on the past productivity, spillover effects, and innovation term of labor-augmenting productivity. u_{it}^2 represents the Hicks-neutral productivity rank of a firm after accounting for the past productivity and spillover effects.

3 Estimation Strategy: A Control Variable Approach

I use the control variable approach by Imbens and Newey (2009) to estimate the production function of the firms. This method constructs control variables to solve for endogeneity problem of the structural models. The key advantage of this method is its ability to identify and estimate models featuring non-separable, multidimensional disturbances. This is the case in my model as I have two unobserved, non-separable productivity disturbances. The standard proxy variable approaches pioneered by Olley and Pakes (1996) accommodate only a single, separable disturbance, rendering them inapplicable in my model.

A control variable is characterized by the conditional distribution function of the endogenous variable conditioning on the instruments. The control variables have two key characteristics: they are strictly monotone in the endogenous variable and independent from the instruments. I will now describe my control variables for labor-augmenting and Hicks-neutral productivity using the assumptions of the model.

I showed that $\tilde{M}_{it} = r_t(\omega_{it}^L)$ in the previous section. Substituting the productivity process

$$\begin{aligned}
\tilde{M}_{it} &= r_t\big(\omega_{it}^L\big) = r_t\Big(g_1\big(\omega_{it-1}^L, \omega_{it}^H, \sum_{j\in I, j\neq i} R_{jt-1}, u_{it}^1\big)\Big) \\
&= r_t\Big(g_1\Big(\bar{r}_t\big(\tilde{M}_{it-1}\big), \bar{s}_t(K_{it-1}, K_{it-1}^{int}, \tilde{M}_{it-1}, M_{it-1}), \sum_{j\in I, j\neq i} R_{jt-1}, u_{it}^1\Big)\Big) \\
&= \tilde{r}_t\Big(W_{it-1}, \sum_{j\in I, j\neq i} R_{jt-1}, u_{it}^1\Big)
\end{aligned}$$

where $\tilde{r}_t(.)$ is an unknown function, and $W_{it-1} = (K_{it-1}, K_{it-1}^{int}, \tilde{M}_{it-1}, M_{it-1})$. Notice that $\tilde{M}_{it}$ is strictly monotone in u_{it}^1 because i) ω_{it}^L is strictly monotone in u_{it}^1 by elasticity of substitution assumption and ii) $\tilde{M}_{it}$ is strictly monotone in ω_{it}^L by the construction of $g_{1t}(.)$ function. Furthermore, from equation 2.7 and timing assumption, u_{it}^1 is independent of W_{it-1} and $\sum_{j\in I, j\neq i} R_{jt-1}$. The proof is in the appendix. These are two conditions required for control variables, i.e. $\tilde{r}_t(W_{it-1}, \sum_{j\in I, j\neq i} R_{jt-1}, u_{it}^1)$ is strictly monotone in u_{it}^1 and u_{it}^1 is independent of W_{it-1} and $\sum_{j\in I, j\neq i} R_{jt-1}$. As u_{it}^1 has already a uniform distribution, the control variable for labor productivity can be defined as

$$u_{it}^1 = F_{\tilde{M}_{it}|W_{it-1}, \sum_{j\in I, j\neq i} R_{jt-1}}\Big(\tilde{M}_{it} \mid W_{it-1}, \sum_{j\in I, j\neq i} R_{jt-1}\Big) \tag{3.1}$$

where $F_{\tilde{M}_{it}|W_{it-1},\sum_{j\in I,j\neq i} R_{jt-1}}(.)$ is the cumulative distribution function of $\tilde{M}_{it}$ conditional on $W_{it-1}, \sum_{j\in I,j\neq i} R_{jt-1}$. That means that conditional on $W_{it-1}, \sum_{j\in I,j\neq i} R_{jt-1}(.)$, material-to-labor ratio in firm i is greater than firm j if and only if $u^1_{it} > u^1_{jt}$. In other words, if two firms have the same past input values, and exposed to a similar sum of knowledge capital the firm has a higher higher material-to-labor ratio if only if it has a higher u^1_{it}.

I can now express ω^L_{it} as a function of observable inputs

$$\begin{aligned}
\omega^L_{it} &= g_1\Big(\omega^H_{it-1}, \omega^L_{it-1}, \sum_{j\in I,j\neq i} R_{jt-1}, u^1_{it}\Big) \\
&= g_1\Big(\bar{r}_t(\tilde{M}_{it-1}), \bar{s}_t(K_{it-1}, K^{int}_{it-1}, \tilde{M}_{it-1}, M_{it-1}), \sum_{j\in I,j\neq i} R_{jt-1}, u^1_{it}\Big) \\
&= c_{1t}\Big(W_{it-1}, \sum_{j\in I,j\neq i} R_{jt-1}, u^1_{it}\Big) \qquad (3.2)
\end{aligned}$$

where $c_{1t}(.)$ is an unknown function. Since I constructed u^1_{it}, the labor productivity can be defined as a function of factor inputs and control variable.

One can, similarly, define the control variable for the Hicks-neutral productivity using the monotonicity assumption,

$$\begin{aligned}
M_{it} &= s_t(K_{it}, K^{int}_{it}, \omega^L_{it}, \omega^H_{it}) \\
&= s_t\Big(K_{it}, K^{int}_{it}, g_1(\omega^L_{it-1}, \omega^H_{it}, \sum_{j\in I,j\neq i} R_{jt-1}, u^1_{it}), g_2(\omega^L_{it-1}, \omega^H_{it}, \sum_{j\in I,j\neq i} R_{jt-1}, u^1_{it}, u^2_{it})\Big)
\end{aligned}$$

Substituting equation 2.4 and 2.6 into ω^L_{it} and ω^H_{it} in the equation above, material demand becomes

$$M_{it} = \tilde{s}_t(K_{it}, K^{int}_{it}, W_{it-1}, \sum_{j\in I,j\neq i} R_{jt-1}, u^1_{it}, u^2_{it})$$

where $\tilde{s}_t()$ is an unknown function and W_{it-1} is the lag values of factor inputs as previously defined. u^2_{it} is independent of K_{it} and K^{int}_{it} due to the timing assumption of capitals. Similarly, its independence from W_{it-1} is due to the inclusion of these variables within the information set of time period $t-1$. Additionally, u^2_{it} is also independent of u^1_{it} from productivity assumption. These conditions collectively satisfy the independence condition for the control variable. A more comprehensive proof is in the appendix.

Furthermore, M_{it} is monotone in u^2_{it} because ω^H_{it} is monotone in u^2_{it} by construction of $c_{2t}(.)$ function and M_{it} is strictly increasing in ω^H_{it} by the material demand assumption. Since two conditions of control variables are satisfied and u_{it}^2 is uniformly distributed, the control variable for Hicks-neutral productivity can be defined as

$$u_{it}^2 = F_{M_{it}|K_{it},K_{it}^{int},W_{it-1},\sum_{j\in I,j\neq i} R_{jt-1}}\Big(M_{it} \mid K_{it}, K_{it}^{int}, W_{it-1}, \sum_{j\in I,j\neq i} R_{jt-1}\Big) \tag{3.3}$$

where $F(.)$ is the CDF of M_{it} conditional on $K_{it}, K_{it}^{int}, W_{it-1}, \sum_{j\in I,j\neq i} R_{jt-1}$. This implies that among firms having the same physical and intangible capital, same past factor inputs, and exposing to the similar aggregate knowledge capital, the one with higher u_{it}^2 exhibit greater material demand.

Using this result, the Hicks-neutral productivity can be constructed as

$$\omega_{it}^H = c_{2t}\big(W_{it-1}, \sum_{j\in I,j\neq i} R_{jt-1}, u_{it}^1, u_{it}^2\big) \tag{3.4}$$

where $c_{2t}(.)$ is an unknown function. This implies that conditional on two control variables, knowledge spillovers from other firms and past factor input variables, there is no variation in the Hicks-neutral productivity.

4 Identification

This section outlines how I identify output elasticity of factor inputs, markup charged by the firms and the effect of knowledge spillovers on firm productivity.

The cost minimization problem, considering material and labor as flexible inputs, leads to the following equation[1]

$$\frac{\theta_{it}^M}{\theta_{it}^L} = \frac{\alpha_{it}^M}{\alpha_{it}^L} \tag{4.1}$$

where θ_{it}^M and θ_{it}^L are output elasticity of material and labor, respectively. α_{it}^M and α_{it}^L denote the revenue share in material and labor, respectively. This relationship indicates that the ratio of output elasticity for the flexible inputs is equal to the revenue share of these inputs. I will leverage this finding to isolate and identify the separate impact of elasticitities.

[1] The proof is in the appendix

Before introducing the identification of output elasticities let's initially define the logarithm of output function as follow

$$y_{it} = f_t(K_{it}, K_{it}^{int}, h_t(\omega_{it}^L L_{it}, M_{it})) + \omega_{it}^H + \epsilon_{it}$$

where $f_t()$ is the logarithm of $F_t()$ function. Given that $h_t()$ function is homogeneous, I make the assumption that it holds homogeneity of degree one. This allows me to factor out L_{it} and define $h_t(\omega_{it}^L L_{it}, M_{it}) = L_{it} h_t(\omega_{it}^L, \tilde{M}_{it})$. Substituting the labor productivity from equation 2.4, $h_t()$ becomes $h_t(\omega_{it}^L L_{it}, M_{it}) = L_{it} h_t(r_t(\tilde{M}_{it}), \tilde{M}_{it}) = L_{it}\bar{h}_t(\tilde{M}_{it})$ where $\bar{h}_t(.)$ is an unknown function. Note that both components of $h_t(.)$ function are dependent on $\tilde{M}_{it}$. This makes infeasible to fully identify all features of the $h_t(.)$ function. However, it will become apparent that $\bar{h}_t(.)$ function will suffice to identify the output elasticities.

Using the $\bar{h}_t()$ function I can rewrite the production function as:

$$y_{it} = f_t(K_{it}, K_{it}^{int}, L_{it}\bar{h}_t(\tilde{M}_{it})) + \omega_{it}^H + \epsilon_{it}$$

Building upon this framework, I will now present the first proposition that is necessary to identify the individual output elasticities.

Proposition 4.1 *The sum of labor and material elasticity is equal to*

$$\theta_{it}^M + \theta_{it}^L = f_{t3}(K_{it}, K_{it}^{int}, L_{it}\bar{h}_t(\tilde{M}_{it}))L_{it}\bar{h}_t(\tilde{M}_{it})$$

where $f_{t3}(.)$ is the derivative of $f_t(.)$ with respect to the third argument.

The proof is in the appendix. The proposition demonstrates that derivative of $f_t(.)$ with respect to its third argument is equivalent to the sum of flexible input elasticities. Importantly, I do not need to identify $h_t(.)$ function as the sum of elasticities do not depend on the derivative of $h_t()$. Instead, the identification of $\bar{h}_t(.)$ is sufficient to determine the sum of elasticities.

Substituting equation 4.1 into the sum of elasticities I can identify the labor and material elasticity as

$$\begin{aligned}\theta_{it}^L &= \frac{\alpha_{it}^L}{\alpha_{it}^L + \alpha_{it}^M} f_{t3}(.)L_{it}\bar{h}_t(\tilde{M}_{it}) \\ \theta_{it}^M &= \frac{\alpha_{it}^M}{\alpha_{it}^L + \alpha_{it}^M} f_{t3}(.)L_{it}\bar{h}_t(\tilde{M}_{it}) \qquad (4.2)\end{aligned}$$

This implies that the sum of material and labor elasticity is distributed based on their respective revenue shares. Since the input shares, represented by α_{it}, are directly observable in the data, there is no need for additional estimation to determine them. Once the sum of elasticities is identified, it becomes straightforward to deduce the individual labor and material elasticities.

Following Loecker and Warzynski (2012) the first order condition of the cost minimization problem yields the expression for markup as follow

$$\mu_{it} = \frac{\theta_{it}^M}{\alpha_{it}^M} = \frac{\theta_{it}^L}{\alpha_{it}^L} \tag{4.3}$$

where μ_{it} stands for the markup rate that firm i charges in year t. This equation indicates that the markup rates can be derived using either the labor elasticity or the material elasticity. When dividing the elasticity of flexible input by its revenue share, the resulting markup rates should be identical. By using the elasticities from equation 4.2, I determine the markup rates at the firm level.

I identify the output elasticity of physical and intangible capital through a similar strategy.

Proposition 4.2 *The output elasticity of physical and intangible capital is*

$$\begin{aligned} \theta_{it}^{int} &= f_{t2}(K_{it}, K_{it}^{int}, L_{it}\bar{h}_t(\tilde{M}_{it}))K_{it}^{int} \\ \theta_{it}^{K} &= f_{t1}(K_{it}, K_{it}^{int}, L_{it}\bar{h}_t(\tilde{M}_{it}))K_{it} \end{aligned} \tag{4.4}$$

where $f_{t2}(.)$ and $f_{t1}(.)$ is the derivative of $f_t(.)$ with respect to the second and first argument, respectively.

This proposition asserts that the output elasticity of physical and intangible capital can be represented as the derivative of $f_t()$ function with respect to the corresponding type of capital. Moreover, $\bar{h}(.)$ is sufficient for it. I do not need to characterize $h_t(.)$ function.

The following proposition shows how I identify the effect of knowledge spillover on the firm productivity.

Proposition 4.3 *The productivity elasticity of knowledge spillover is*

$$\theta_{it}^{spillover} = \frac{\partial c_{2t}\big(W_{it-1}, \sum_{j\in I, j\neq i} R_{jt-1}, u_{it}^1, u_{it}^2\big)}{\partial \sum_{j\in I, j\neq i} R_{jt-1}} \frac{\sum_{j\in I, j\neq i} R_{jt-1}}{c_{2t}(.)} \tag{4.5}$$

The effect of knowledge spillover on firm productivity corresponds to the derivative of the Hicks-neutral productivity function, generated from the control variables, with respect to the sum of knowledge capital.

5 Data and Empirical Model

This section describes the data used for estimating the model and presents the estimation strategy.

5.1 Data

I use the U.S. Compustat data to measure firm-level intangible capital and other variables from financial statements, including sales, materials, number of employees, physical capital, and industry classification. The Compustat sample covers all public firms in the US from 1975 to 2020. Following the sampling procedures in the literature, I exclude financial firms (SIC codes 4900 - 4999), utilities (SIC codes 6000 - 6999), and government (SIC code 9000 and above). I also exclude firms with missing or negative sales, capital, employment, or SG&A expenditure, and very small firms with a number of employees less than 10. I trim the variables at the 1% level.

I measure physical capital as the value of deflated property, plant, and equipment. Labor is the number of employees and output is the deflated net sales. I calculate materials as deflated cost of goods sold. All the variables is deflated using the BEA industry-specific price deflators. I now describe how to construct the firm-level intangible capital.

Measurement of Intangible Capital. I construct the intangible capital at the firm level as outlined in Ewens et al. (2019) (along with insights from other studies such as Lev and Radhakrishnan (2005), Eisfeldt and Papanikolaou (2014)). Intangible capital consists of two components: knowledge capital and organizational capital.

I measure knowledge capital based on Research and Development (R&D) expenses. These R&D investments are recorded as flow variables in Compustat. I convert them into a stock variable using the perpetual inventory method as follows

$$R_{it} = (1 - \delta_{R\&D})R_{it-1} + R\&D_{it} \tag{5.1}$$

where R_{it} is the knowledge capital of firm i in year t, $R\&D_{it}$ is the firm i's R&D investment in year t, and $\delta_{R\&D}$ is the industry-specific R&D depreciation rates based on the estimates of Ewens et al. (2019). I initialize the value of R_{i0} as zero.

I construct organizational capital using Selling, General, and Administrative Expenses (SG&A). SG&A includes a variety of expenses related to various operating activities. This approach requires to represent a portion of the total SG&A as organizational investment (Lev and Radhakrishnan (2003)). To capitalize on the organizational investments, I similarly adopt the perpetual inventory method, as follows

$$O_{it} = (1 - \delta_{SG\&A})O_{it-1} + \gamma SG\&A_{it} \tag{5.2}$$

where O_{it} is the organizational capital stock of firm i in year t, $SG\&A$ is the selling general and administrative spending of firm i in year t, and γ corresponds to the industry-specific proportion of SG&A expenses that are allocated to organizational activities. I use industry-specific estimates of $\delta_{SG\&A}$ and γ following Ewens et al. (2019). I set B_{i0} as zero.

Hence, the intangible capital stock at the firm level can be described as:

$$K_{it}^{int} = R_{it} + O_{it} \tag{5.3}$$

where K_{it}^{int} is the firm i's intangible capital stock in year t. This equation highlights that intangible capital is the sum of both knowledge and organizational capital.

5.2 Estimation Strategy

This section outlines the estimation procedure for the model. I estimate the production functions for different industries over time, categorizing firms based on Fama-French 5 industry classification since I use the parameters of intangible capital estimated for the Fama-French 5 industries by Ewens et al. (2019). Due to a small sample size in some sectors I use 7-year rolling windows.

I defined the logarithm of production function in the identification section as follow

$$y_{it} = f_t(K_{it}, K_{it}^{int}, L_{it}\bar{h}_t(\tilde{M}_{it})) + \omega_{it}^H + \epsilon_{it}$$

I substitute the Hicks-neutral productivity that I constructed using the control variables (equation 3.4) into the production function

$$y_{it} = f_t(K_{it}, K_{it}^{int}, L_{it}\bar{h}_t(\tilde{M}_{it})) + c_{2t}\big(W_{it-1}, \sum_{j\in I, j\neq i} R_{jt-1}, u_{it}^1, u_{it}^2\big) + \epsilon_{it} \tag{5.4}$$

where $E[\epsilon_{it} \mid W_{it}, W_{it-1}, \sum_{j\in I, j\neq i} R_{jt-1}] = 0$ since I have control variables dealing with the endogeneity problem. I can estimate this estimation equation by minimizing the sum of squared residuals. But, this is not the only moment condition I have. Using the predetermined assumption of capital I can construct my other moment condition. The error terms in my productivity assumption was non-separable. Following the The proxy variable approach used mostly in the literature I can define the productivity as

$$\omega_{it}^H = c_{3t}(\omega_{it-1}^H, \omega_{it-1}^L, \sum_{j\in I, j\neq i} R_{jt-1}) + \upsilon_{it} \tag{5.5}$$

where $c_{3t}(.)$ is an unknown function, and υ_{it} is separable from $c_{3t}(.)$ with $E[\upsilon_{it}|I_{t-1}] = 0$. This is different from the productivity assumption in the previous section because the innovation terms, u_{it}^1, and u_{it}^2, were non-separable and independent whereas υ_{it} is separable and mean-independent. The separability assumption is mostly used in the production function estimation.

Using the model's results and assumptions I can write the Hicks-neutral productivity as $\omega_{it}^H = \bar{c}_{3t}(W_{it-1}, \sum_{j\in I, j\neq i} R_{jt-1}) + \upsilon_{it}$ for an unknown function of $\bar{c}_{3t}(.)$. The production function can be written as

$$y_{it} = f_t(K_{it}, K_{it}^{int}, L_{it}\bar{h}_t(\tilde{M}_{it})) + \bar{c}_{3t}\big(W_{it-1}, \sum_{j\in I, j\neq i} R_{jt-1}\big) + \upsilon_{it} + \epsilon_{it} \tag{5.6}$$

where $E[\upsilon_{it} + \epsilon_{it}|I_{t-1}] = 0$. Since physical and intangible capital is predetermined variables I should have $E[\upsilon_{it} + \epsilon_{it}|K_{it}] = 0$ and $E[\upsilon_{it} + \epsilon_{it}|K_{it}^{int}] = 0$. Combining with the moment condition 5.4, I construct my objective function to minimize as follows

$$\frac{1}{N}\sum_i \epsilon_{it}^2 + \left(\frac{1}{N}\sum_i (\epsilon_{it} + \upsilon_{it})K_{it}\right)^2 + \left(\frac{1}{N}\sum_i (\epsilon_{it} + \upsilon_{it})K_{it}^{int}\right)^2 \tag{5.7}$$

In order to minimize the objective function I need to have the control variables, the functions of $f_t(.), \bar{h}_t(.)$, $c_{2t}(.)$, and $\bar{c}_{3t}(.)$. First, I estimate u^1_{it} and u^2_{it} control variables using the equations 3.1 and 3.3. I use logistic regression to estimate them. It's important to note that the outcome variable is not discrete in both cases. I partition them into 500 grids and I estimate the CDFs at those points with third-order polynomials using the logistic regression. I then interpolate other points. Second, I first approximate $\bar{h}_t()$ using third-order polynomials. Given $\bar{h}_t()$, I approximate $f_t()$, $c_{2t}()$ and $\bar{c}_{3t}()$ functions with second-order polynomials. I, then, minimize the moment condition in equation 5.7. After having the estimates of those functions, I calculate output elasticities, markup and the spillover effects using the propositions outlined in the identification section. I perform 100 bootstraps to estimate standard errors for the identified outcomes, treating firms as independent and resampling them with replacement.

6 Results

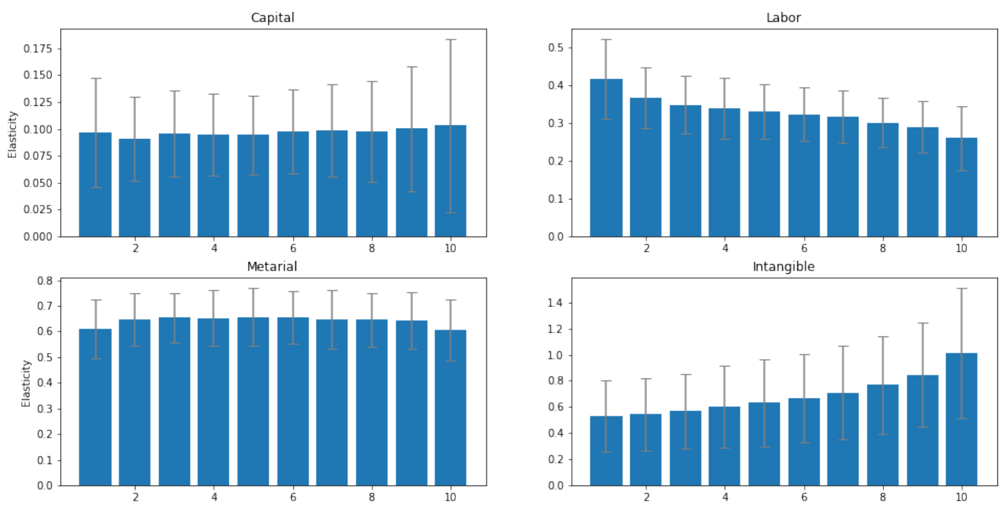
I begin by presenting the output elasticity of factor inputs. These results indicate that the intangible elasticity increases with firm size, suggesting scalability of intangible assets. Next, I analyze the impact of external knowledge on firm productivity. Finally, I examine the markup rates.

6.1 Output Elasticities

Figure 1 illustrates the output elasticity of capital, intangible capital, material, and labor at each firm-size decile. Firms are ranked by their sales within sector-year level, with 10 representing the largest firms and 1 denoting the smallest ones. I then take the average factor elasticities within each decile across sector year. The figure reveals that as firm size increases, intangible elasticity rises, while labor elasticity decreases. Material and capital elasticity, on the other hand, shows relatively minor fluctuations. These findings are consistent with the existing literature and emphasize the necessity of non-parametric estimation methods to capture the heterogeneous cross-firm output elasticity.

Since intangible assets lack a physical presence, firms can efficiently replicate them in their multiple production processes, yielding greater benefits. I use firm size as a proxy for the production process of the firms. The figure shows that with the rise in firm size benefit to intangibles also increases. Specifically, when small firms invest 1% in intangible assets their output increases around 0.5%, whereas the largest firms reach levels of around 1.05%. This shows that large firms benefit more from intangible assets, suggesting the scalability feature of intangible assets.

Figure 1: Output Elasticity of Capital, Labor, Material, and Intangibles by Firm Size

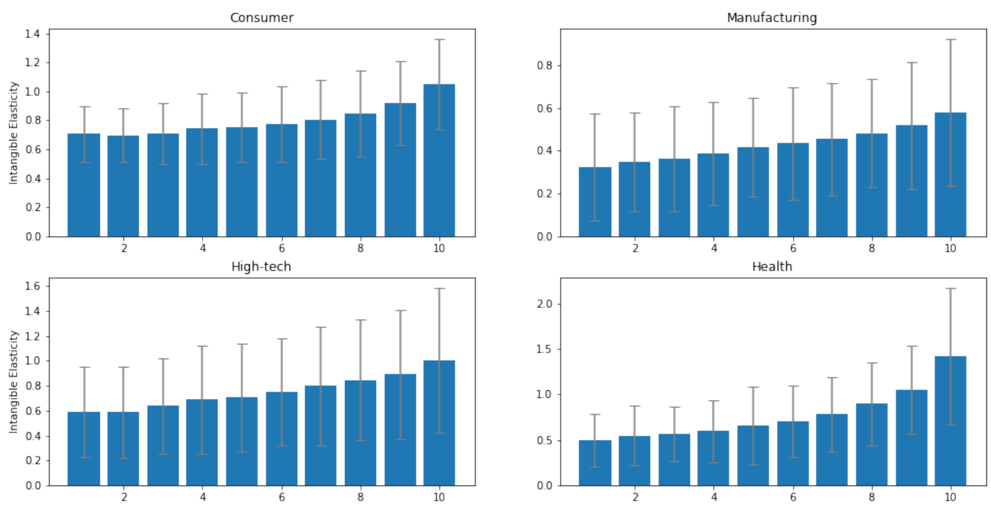


Note: This figure shows output elasticities by firm decile. The average elasticity for each decile within an industry year is estimated first, then these estimates are averaged across industry-year bins. The error bars indicate 95% confidence intervals calculated using bootstrap (100 iterations)

The returns on intangible assets can indeed exhibit sector-specific disparities, and the observed scalability trend may be contingent upon sectoral characteristics. For instance, high-tech firms possess the ability to leverage their intangible assets across multiple facets of production processes, given the inherently intangible nature of the sector. In contrast, manufacturing firms may not share the same level of versatility in this regard. Consequently, the documented escalation in intangible elasticity with firm size could be predominantly attributed to the high-tech sector, with a less pronounced effect in manufacturing.

Figure 2 illustrates that, across all sectors, the return on intangibles tends to increase with firm size, albeit to varying degrees. For instance, in the health sector, the largest firms exhibit an elasticity of 1.5, indicative of substantial scalability in intangible capital utilization. In contrast, in the manufacturing sector, this elasticity stands at 0.5, reflecting a relatively more modest return on intangible assets. Nevertheless, it is noteworthy that even within the manufacturing sector, there is still an observable enhancement in the returns on intangible assets as firm size expands. The return for the smallest firms in manufacturing is around 0.3, whereas the elasticity for the largest firms is around 0.5. Figure 2 also shows the elasticities in the consumer and high-tech sectors, in addition to the health and manufacturing sectors. All these sectors clearly exhibit a similar pattern: a positive association between firm size and the return on intangible assets, implying that larger firms, regardless of sector, tend to capitalize more efficiently on intangible resources.

Figure 2: Output Elasticity of Intangible Capital by Firm Size and Sector



Note: This figure shows output elasticities within a sector by firm decile. The average elasticity for each decile within an industry-year is estimated first, then these estimates are averaged across years within a sector. The error bars indicate 95% confidence intervals calculated using bootstrap (100 iterations)

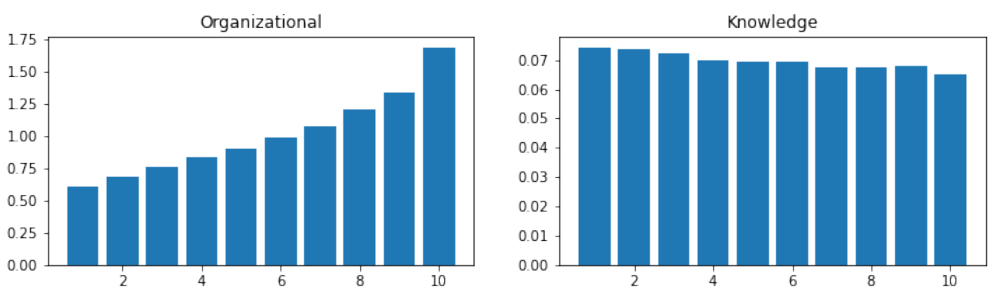
The sector heterogeneity may not be the only factor behind the intangible returns. Different intangible assets might have different scalability levels. As I introduced in the data section, intangible assets encompass both knowledge and organizational capital, each potentially exhibiting distinct scalability characteristics. To differentiate the impacts of these components, I incorporate knowledge and organizational capital separately into the production function, rather than defining intangible capital as a sum of knowledge and organizational capital. The modified production function takes the form of

$$Y_{it} = F(K_{it}, R_{it}, O_{it}, h_t(\omega_{it}^L L_{it}, M_{it}))exp(\omega_{it}^H)exp(\epsilon_{it})$$

where R_{it} represents the knowledge capital and O_{it} is the organizational capital. Applying a similar estimation procedure as outlined in the Estimation section, I compute the output elasticity of both knowledge and organizational capital.

Figure 3 reveals that the returns attributed to intangible capital are predominantly driven by organizational capital, with knowledge capital demonstrating considerably lower elasticity. The return on organizational capital varies between 0.5 and 1.6 across different firm sizes, while knowledge capital maintains a relatively stable elasticity of approximately 0.07 across firms. Furthermore, the elasticity of organizational capital exhibits an increasing relationship with firm size, whereas knowledge capital maintains a consistent pattern across firm sizes. These results suggest that the scalability feature of intangible capital is primarily propelled by organizational capital instead of knowledge capital. It's important to note that these findings are specific to the short-term effects on output. The long-term dynamics may differ, and knowledge capital could potentially yield higher returns over time as it accumulates and contributes to a firm's competitive advantage. These results shed light on the current impact of knowledge and organizational capital on production, illustrating how much output increases when a firm augments its knowledge or organizational capital by a given percentage.

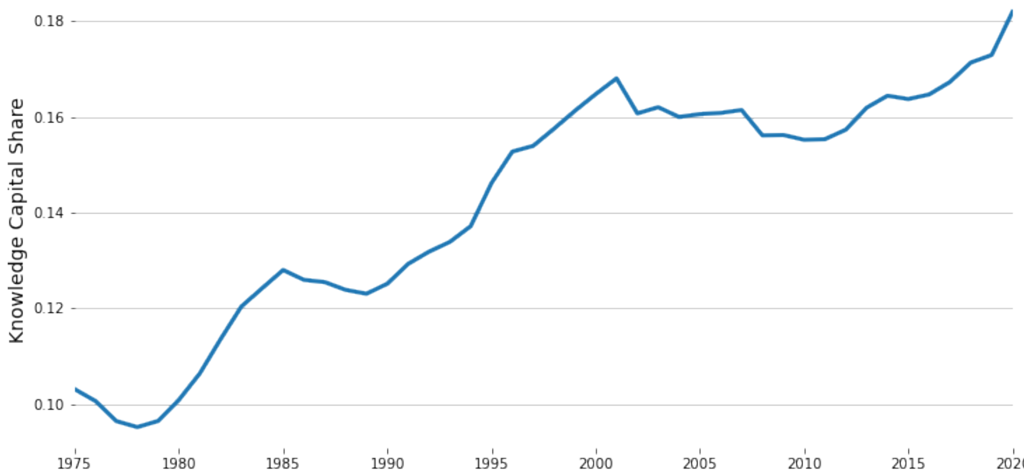
Figure 3: Output Elasticity of Intangible Components



Note: This figure shows output elasticities of knowledge and organizational capital by firm decile. The average elasticity for each decile within an industry-year is estimated first, then these estimates are averaged across the year within industry-year bins.

The substantial short-run return on organizational capital may indeed provide an explanation for why firms tend to allocate a significant portion of their investments to organizational capital rather than knowledge capital. As depicted in Figure 4, the share of knowledge capital constitutes only around 15% of the total intangible assets, while the share of organizational capital accounts for approximately 85%. This allocation remains consistent over time, despite a slight uptick in the share of knowledge capital.

Figure 4: Knowledge Capital Share in Intangible Capital



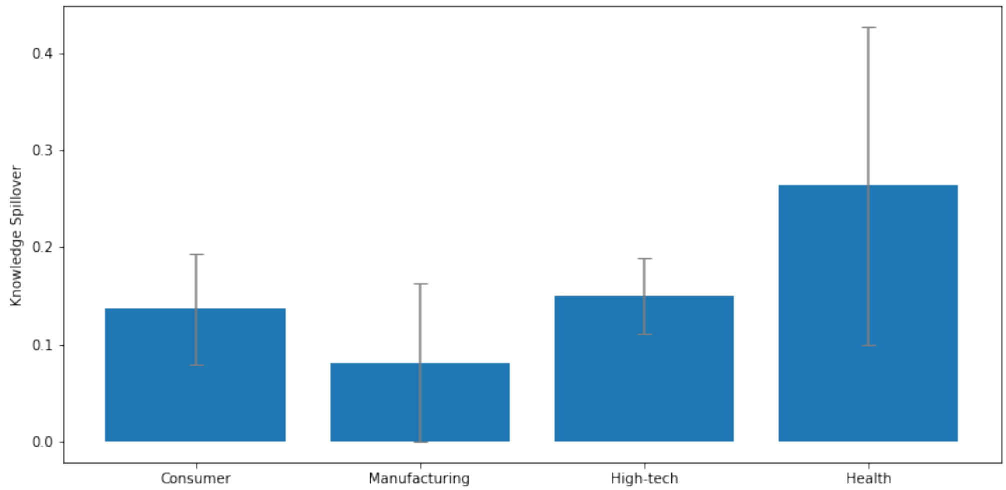
Note: This figure shows knowledge capital share in total intangible capital. The share of knowledge capital is averaged across firms within a year.

6.2 Knowledge Spillovers

This section presents the influence of knowledge spillovers on firm productivity, estimating the firm-level productivity elasticity in response to external knowledge. The findings reveal a consistently positive and statistically significant impact of external knowledge on firm productivity, indicating that firms gain valuable insights and benefits from their industry counterparts. Nonetheless, these benefits exhibit notable sectoral variations.

Figure 5 represents the impact of knowledge spillovers across different sectors. It is evident that firms across various sectors derive advantages from the knowledge stock of their rivals. Notably, firms in the health sector experience the most substantial gains, while those in manufacturing witness comparatively modest benefits. High-tech and consumer-oriented firms also exhibit significant enhancements in their productivity stemming from their competitors' knowledge. However, it's noteworthy that the manufacturing sector, while experi-

Figure 5: Knowledge Spillover by Sector

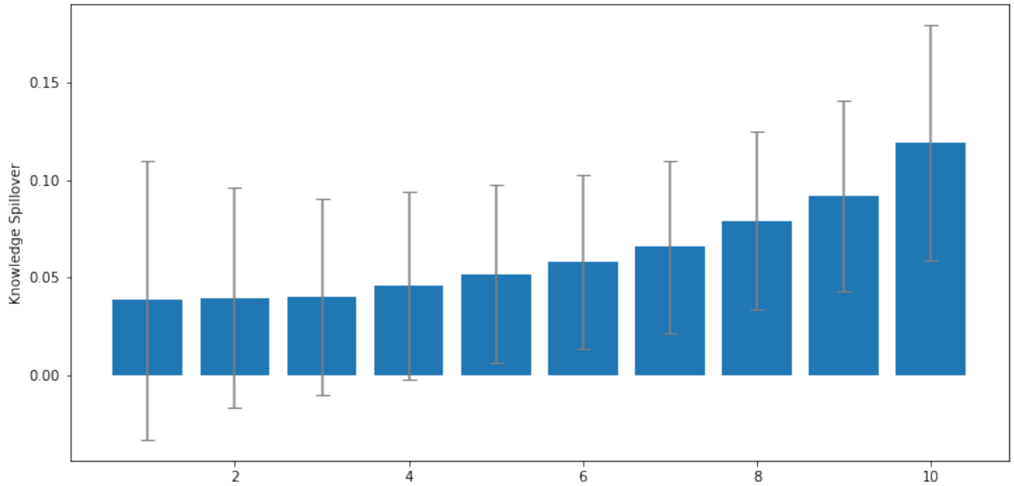


Note: This figure shows the knowledge spillover elasticity computed as a productivity elasticity of external knowledge in the industry. The firm-level spillover elasticity is averaged across years within a sector. The error bars indicate 95% confidence intervals calculated using bootstrap (100 iterations)

encing a slight positive effect, fails to achieve statistical significance at the 95% confidence interval. Intriguingly, despite the health sector's leading position in benefiting from knowledge spillovers, it demonstrates a wider confidence interval compared to the high-tech and consumer sectors. This variance can be attributed to the pronounced heterogeneity within the health sector, where certain firms experience substantial gains from knowledge spillovers, while others do not benefit to the same extent.

While sectoral heterogeneity plays a significant role in benefiting from knowledge spillovers, it is not the sole source of heterogeneity, as evident from the high confidence intervals observed in the health and manufacturing sectors. To provide a more comprehensive understanding of this phenomenon, I examine the impact of firm size on the productivity elasticity of the external knowledge. Figure 6 offers a detailed perspective on the influence of knowledge spillovers on firm productivity, categorizing firms based on their size. The figure unveils a discernible pattern: larger firms tend to reap more substantial benefits from the existing knowledge generated within their respective industries. In essence, there exists a positive

Figure 6: Knowledge Spillover by Firm Size



Note: This figure shows the productivity elasticity of external knowledge capital by firm decile. The average elasticity for each decile within an industry-year is estimated first, then these estimates are averaged across industry-year bins. The error bars indicate 95% confidence intervals calculated using bootstrap (100 iterations)

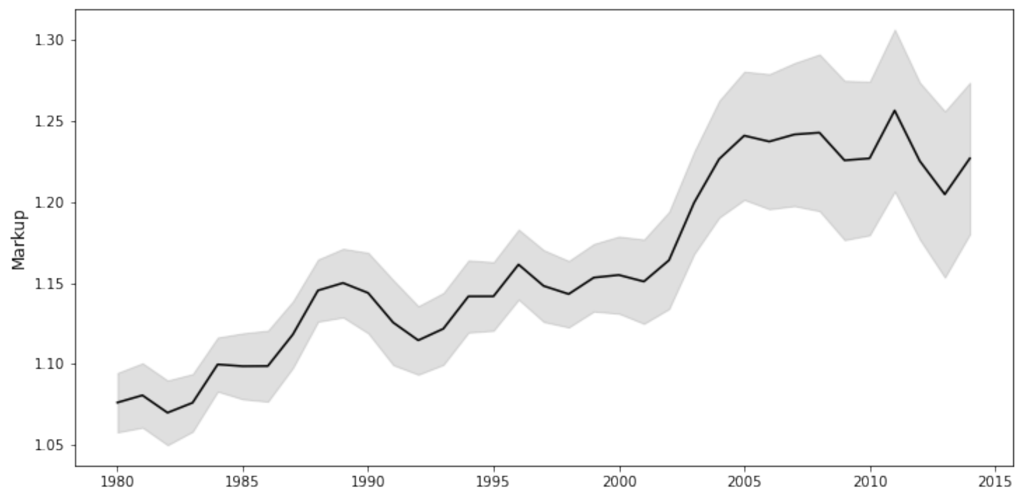
correlation between firm size and the impact of knowledge spillovers on firm productivity. The small firms appear to derive relatively lower and statistically insignificant benefits from the knowledge generated by their industry rivals. This relationship highlights the crucial role of firm size in determining the ability to effectively utilize external knowledge.

6.3 Markup

The scalability feature of intangible assets enables large firms to grow faster, which in turn can lead to increased market concentration in industries. The best measure to assess market concentration is the markup rates charged by firms. This section explores the markup rates and their relationship to intangible assets.

Figure 7 illustrates the average markup charged by a public firm over time. The upward trend in markup rates has persisted since the 1980s, despite occasional fluctuations. In the

Figure 7: Average Markup

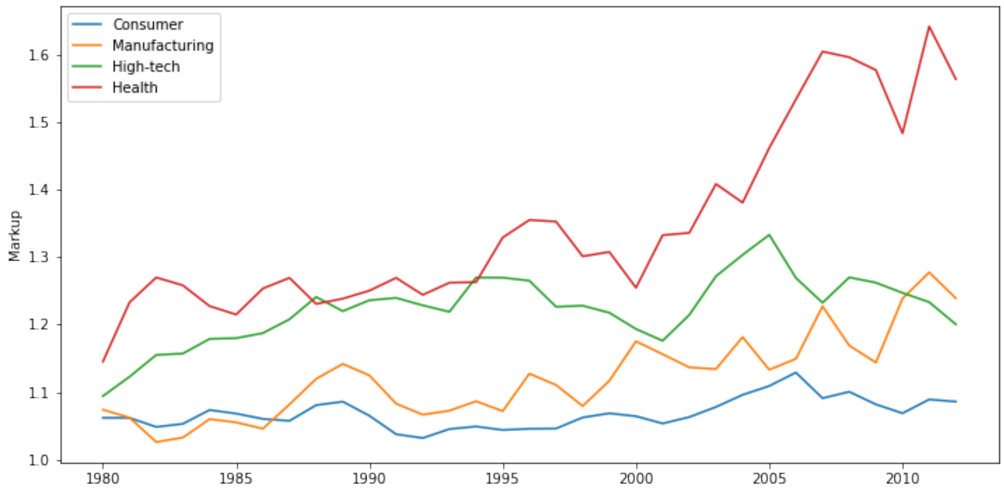


Note: This figure shows the annual average markup rate. The estimated firm-level markups are averaged across firms within a year. The error bars indicate 95% confidence intervals calculated using bootstrap (100 iterations)

early 1980s, the average markup stood at approximately 1.07, while in recent years, it has climbed to around 1.25. This aligns with the existing literature that has also reported a consistent rise in markups over the past few decades. My markup rates are closer to the markups estimated in De Loecker et al. (2020) when they control for selling, general, and administrative expenses in their production function.

Figure 8 displays the average markup rates across the Fama-French 5 industries. An increasing trend is evident in all sectors, with the health and high-tech sectors emerging as the primary drivers behind this markup rise. In the early 1980s, these sectors exhibited similar rates with minor variations. However, over time, high-tech and particularly the health sector diverged, with firms in these sectors charging significantly higher markups compared to those in consumer and manufacturing industries. In recent years, markups in the health sector have reached levels as high as 1.6. The high-tech markup levels were adversely affected in the early 2000s due to the dot-com bubble, and there were low markups around the 2008 crisis. However, in general, high-tech displayed a similar pattern to the

Figure 8: Average Markup for Fama-French 5 Industries



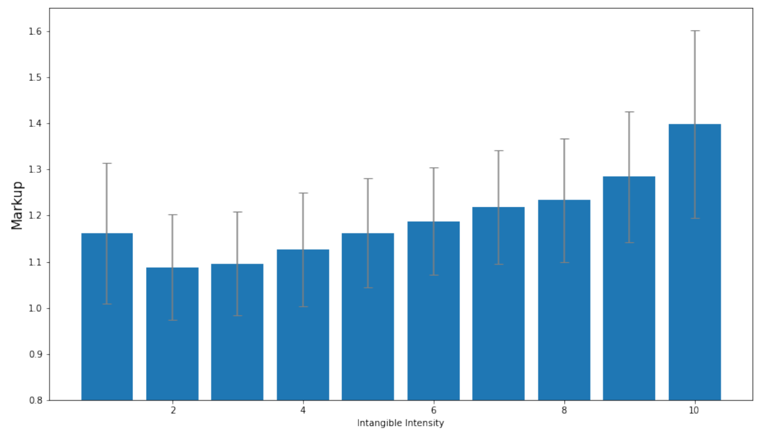
Note: This figure shows the annual average markup rates for different sectors. The estimated firm-level markups are averaged across firms within a year for each sector. The error bars indicate 95% confidence intervals calculated using bootstrap (100 iterations)

health sector. Notably, when considering these sectors, intangible capital emerges as the primary component of their production. This observation suggests a potential link between the firms' intangible intensity and the markup levels they charge.

Figure 9 displays the average markup rates charged by firms based on their intangible intensity. I rank firms by their intangible intensity within each sector and year. I calculate the average markup rates for each decile within a sector and year, then average across sectors and years within each decile. The figure reveals that low intangible intense firms charge approximately 1.1, while the most intangible-intensive firms charge around 1.35. This pattern holds across increasing levels of intangible intensity, indicating a positive relationship between markup levels and the intangible intensity of firms. This result may be attributed to the scalability feature of intangibles. As intangible assets enable large firms to grow more rapidly, increased investments in intangible assets may grant them market power within their sector, allowing them to charge higher markups.

To comprehensively account for the various factors that influence markup rates, I conduct

Figure 9: Average Markup for Fama-French 5 Industries



Note: This figure shows markups by firm decile ranked by intangible intensity. Intangible intensity is defines as $\frac{K_{it}^{int}}{K_{it}+K_{it}^{int}}$. 1 represents the least intangible intense and 10 is the most intangible intense firms. The average elasticity for each decile within an industry-year is estimated first, then these estimates are averaged across industry-year bins.

a regression analysis that incorporates several key firm control variables. These include the firm's market share within its industry, age, profit level, leverage, and company size, alongside firm, year, and sector-time fixed effects. By integrating these control variables into the analysis, I aim to gain a deeper understanding of the relationship between markup rates and intangible intensity, recognizing that intangible intensity alone may not be the sole determinant of markup rates.

Table 1 reveals a significantly positive association between intangible intensity and markup rates, even after controlling for various firm-level control variables. The first column of the table includes control variables but no fixed effects. In this specification, a 1% increase in intangible intensity corresponds to an 8.2% increase in markup rates. However, when firm fixed effects are introduced in the second column, this elasticity decreases to 3.6%. With the inclusion of both firm and time fixed effects in the third column, the elasticity becomes 3.47%. Finally, in the fourth column, which incorporates firm and sector-year fixed effects, the elasticity stands at 3%. These results indicate that firms with higher intangible intensity tend to exhibit higher markup rates. This suggests that the recent surge in intangible capital can potentially be a contributing factor to the recent increase in market concentration, as firms have progressively increased their intangible intensity over the last decades.

Table 1: Relationship between Markup and Intangible Intensity

	Markup	Markup	Markup	Markup
Intangible Intensity	0.0823***	.0369***	.0347***	.0305***
	(0.001)	(.002)	(.002)	(0.001)
Controls	yes	yes	yes	yes
Firm FE	no	yes	yes	yes
Year FE	no	no	yes	no
Sector-year FE	no	no	no	yes
Adjusted R2	0.247	0.815	0.826	0.842

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

7 Robustness Check

7.1 Controlling for the Product Demand

One concern about the Compustat data is that the input and output variables are measured in revenue. In other words, the output in our regression becomes PQ where P is the product price and Q is the quantity produced. Similarly, intangible capital is $K_{it}^{int} = P_t^{int} Q_{it}^{int}$, $K_{it} = P_t^k Q_{it}^k$ and $M_{it} = P_t^m Q_{it}^m$ where P_t^{int}, P_t^k, P_t^m are the price of intangible capital, capital and material, respectively. Note that as the input market is assumed to be perfectly competitive, the input price will be constant across the firms. By deflating the factor inputs, I control input prices, ensuring that the quantity of factor inputs is measured. However, since the output market is imperfectly competitive, even if I deflate the sales, the output price variation will not be captured. Thus, after deflating the variables the estimation becomes

$$\frac{R_{it}}{P_t} = \frac{P_{it}Q_{it}}{P_t} = F_t(Q_{it}^k, Q_{it}^{int}, h_t(\omega_{it}^L L_{it}), Q_{it}^m) exp(\omega_{it}^H) exp(\epsilon_{it}) \tag{7.1}$$

where R_{it} the revenue of the firm i in year t.

I follow Klette and Griliches (1996) to control for the price variation among the firms. Under the CES demand aggregator, the demand for each product is

$$Q_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\sigma} Q_t exp(\epsilon_{it}^d) \tag{7.2}$$

where σ is the elasticity of substitution among the products, Q_t is the aggregate demand in the sector, and ϵ_{it}^d is an idiosyncratic demand shock. Rearranging the equation 7.2 the deflated output price becomes

$$\frac{P_{it}}{P_t} = \left(\frac{Q_{it}}{Q_t}\right)^{-1/\sigma} exp((\epsilon_{it}^d)^{1/\sigma}) \tag{7.3}$$

Substituting the equation 7.3 into the equation 7.1, the logarithm of the equation 7.1 becomes

$$r_{it} = (1 - \frac{1}{\sigma}) f_t(Q_{it}^k, Q_{it}^{int}, h_t(\omega_{it}^L L_{it}), Q_{it}^m) + (1 - \frac{1}{\sigma})\omega_{it}^H + \frac{1}{\sigma} q_t + \frac{1}{\sigma}\epsilon_{it}^d + (1 - \frac{1}{\sigma})\epsilon_{it} \tag{7.4}$$

where r_{it} is the logarithm of the deflated revenue, q_{it} is the logarithm of Q_{it} and q_t is the logarithm of Q_t. This equation states that when revenue is used for the production function estimation, the coefficient estimates become biased as we multiply the production function by $1 - \frac{1}{\sigma}$ and we have additional control variable which is the total industry demand. In order to isolate the interaction between demand and production coefficients, I estimate the equation 7.4 as follows. I proceed similar estimation strategy as described in the estimation strategy section. I, additionally, include the size of sector q_t as an additional control variable. I first obtain the estimate for σ from this regression and then correct the elasticities by $1 - \frac{1}{\sigma}$.

Table 2: Relationship between Markup and Intangible Intensity

	Markup	Markup	Markup	Markup
Intangible Intensity	0.108***	.092***	.069***	.067***
	(0.001)	(.008)	(.002)	(0.001)
Controls	yes	yes	yes	yes
Firm FE	no	yes	yes	yes
Year FE	no	no	yes	no
Sector-year FE	no	no	no	yes
Adjusted R2	0.205	0.815	0.846	0.872

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 2 demonstrates the robustness of my results after accounting for price variations across firms. In the first column, controlling for firm characteristics reveals that firms investing more in intangible assets charge a 10% higher markup. The second column introduces firm fixed effects, while the third column includes both firm and time fixed effects. The last column incorporates firm and sector-year fixed effects. Across all specifications, the consistent trend emerges: as a firm's intangible intensity increases, the markup it charges also rises. This finding holds true even when accounting for various fixed effects, highlighting the robust relationship between intangible investments and markup rates. Thus, my results are robust even after controlling for the price variation modeling the demand side.

8 Conclusion

This study estimates returns to intangible capital heterogeneity using nonparametric production functions. I find that large firms tend to have higher returns from intangible assets compared to their smaller counterparts, indicative of a scalability feature associated with intangibles. Furthermore, while sectoral differences exist in the benefits of intangible assets, the pattern of increasing returns with firm size is consistent across all sectors. Moreover, I demonstrate that firms improves their productivity by leveraging the knowledge stock of their rivals. Firms in the health sector benefit the most from their competitors' knowledge stock, while manufacturing firms do not share the same advantage. Finally, the positive correlation between markup rates and intangible intensity offers a potential explanation for the recent rise in market concentration within the US economy.

Appendix

A Proofs

A.1 Proof of Equation 2.3

The firms minimize their flexible inputs as physical and intangible capital are predetermined variables. They optimize their level to produce at least planned output, $\bar{Y}_{it}$. The cost minimization problem, then, becomes:

$$\min_{L_{it},M_{it}} \left\{ p_t^L L_{it} + p_t^M M_{it} : \mathbb{E}[F_t(K_{it}, K_{it}^{int}, h_t(\omega_{it}^L L_{it}, M_{it}))exp(\omega_{it}^H)exp(\epsilon_{it})|\mathcal{I}_{it}] \geq \bar{Y}_{it} \right\} \quad \text{(A.1)}$$

By the timing assumption, as capitals and ω_{it} and ω_{it}^L are known, the cost problem turns out

$$\min_{L_{it},M_{it}} \left\{ p_t^L L_{it} + p_t^M M_{it} : F_t(K_{it}, K_{it}^{int}, h_t(\omega_{it}^L L_{it}, M_{it}))exp(\omega_{it}^H)\mathbb{E}[exp(\epsilon_{it})\mathcal{I}_{it}] \geq \bar{Y}_{it} \right\}$$

This problem can be written as the choice of effective labor rather than labor. $\bar{L}_{it} = L_{it}\omega_{it}^L$ is defined as the effective labor. Defining quality adjusted wages $p_{it}^L = \frac{p_t^L}{\omega_{it}^L}$ I can now rewrite the problem in terms of effective labor, $\bar{L}_{it}$

$$\min_{\bar{L}_{it},M_{it}} \left\{ \bar{p}_{it}^L \bar{L}_{it} + p_t^M M_{it} : F_t(K_{it}, K_{it}^{int}, h_t(\bar{L}_{it}, M_{it}))exp(\omega_{it}^H)\mathbb{E}[exp(\epsilon_{it})] \geq \bar{Y}_{it} \right\} \quad \text{(A.2)}$$

These two problems are equivalent as I just redefine the optimization problem with effective labor and labor productivity ω_{it}^L is known by the firms at time t. For easy notation, defining $\tilde{Y}_{it} = \frac{\bar{Y}_{it}}{\mathbb{E}[exp(\epsilon_{it}\mathcal{I}_{it}]exp(\omega_{it}^H)}$ the cost problem becomes

$$C_t(K_{it}, K_{it}^{int}, \bar{Y}_{it}, p_{it}^L, p_t^M, \omega_{it}^H) = \min_{\bar{L}_{it},M_{it}} \left\{ \bar{p}_{it}^L \bar{L}_{it} + p_t^M M_{it} : F_t(K_{it}, K_{it}^{int}, h_t(\bar{L}_{it}, M_{it})) \geq \tilde{Y}_{it} \right\}$$

As $F_t()$ is a homogeneous function in $h_t()$, I can invert $F_t()$ function in $h_t()$. As the objective function is linear I can reconstruct the problem as

$$C_t(K_{it}, K_{it}^{int}, \bar{Y}_{it}, p_{it}^L, p_t^M, \omega_{it}^H) = \min_{\bar{L}_{it}, M_{it}} \left\{ \bar{p}_{it}^L \bar{L}_{it} + p_t^M M_{it} : h_t(\bar{L}_{it}, M_{it}) \geq F_t^{-1}(K_{it}, K_{it}^{int}, \tilde{Y}_{it}) \right\}$$

As $K_{it}, K_{it}^{int}, \omega_{it}^H$ and $\bar{Y}_{it}$ is given at time t, the problem can be rewritten as

$$C_t(K_{it}, K_{it}^{int}, \bar{Y}_{it}, p_{it}^L, p_t^M, \omega_{it}^H) = F_t^{-1}(K_{it}, K_{it}^{int}, \tilde{Y}_{it}) \min_{\bar{L}_{it}, M_{it}} \left\{ \bar{p}_{it}^L \bar{L}_{it} + p_t^M M_{it} : h_t(\bar{L}_{it}, M_{it}) \geq 1 \right\}$$

Therefore, the cost minimization problem pins down to

$$C_t(K_{it}, K_{it}^{int}, \bar{Y}_{it}, p_{it}^L, p_t^M, \omega_{it}^H) = C_{1t}(K_{it}, K_{it}^{int}, \bar{Y}_{it}, \omega_{it}^H) C_{2t}(p_{it}^L, p_t^M)$$

where I redefine $F_t^{-1}()$, $C_{1t}(K_{it}, K_{it}^{int}, \bar{Y}_{it}, \omega_{it}^H) = F_t^{-1}(K_{it}, K_{it}^{int}, \tilde{Y}_{it}) \min_{\bar{L}_{it}, M_{it}}$. C_{2t} is the results of minimization problem. Since p_{it}^L and p_t^M are the only state variables left under the cost minimization problem, the minimization problem will only be a function of them. In other words, minimization part will be $C_{2t}(p_{it}^L, p_t^M)$.

Using Shephard's Lemma, the firms' optimal material and labor demand will be a derivative of the minimized cost function with respect to the corresponding prices, i.e.

$$\begin{aligned} M_{it} &= \frac{\partial C_t(K_{it}, K_{it}^{int}, \bar{Y}_{it}, p_{it}^L, p_t^M, \omega_{it}^H)}{\partial p_t^M} = C_{1t}(K_{it}, K_{it}^{int}, \bar{Y}_{it}, \omega_{it}^H) \frac{\partial C_{2t}(p_{it}^L, p_t^M)}{\partial p_t^M} \\ \bar{L}_{it} &= \frac{\partial C_t(K_{it}, K_{it}^{int}, \bar{Y}_{it}, p_{it}^L, p_t^M, \omega_{it}^H)}{\partial p_{it}^L} = C_{1t}(K_{it}, K_{it}^{int}, \bar{Y}_{it}, \omega_{it}^H) \frac{\partial C_{2t}(p_{it}^L, p_t^M)}{\partial p_{it}^L} \end{aligned}$$

As the first part of the cost minimization problem, $C_{1t}(K_{it}, K_{it}^{int}, \bar{Y}_{it}, \omega_{it}^H)$ does not depend on flexible input prices, I factor out when taking derivatives. I will only need to take the derivative of $C_{2t}(p_{it}^L, p_t^M)$ to find the flexible input demand. Taking the ratio between material and labor demand, the equations above becomes

$$\frac{M_{it}}{\bar{L}_{it}} = \frac{\partial C_{2t}(p_{it}^L, p_t^M)/\partial p_t^M}{\partial C_{2t}(p_{it}^L, p_t^M)/\partial p_{it}^L}$$

The material-to-effective labor ratio only depends on the factor input prices. Replacing the effective labor $\bar{L}_{it} = L_{it}\omega_{it}^L$ into the equation, material-to-labor ratio becomes

$$\frac{M_{it}}{L_{it}} = \frac{\partial C_{2t}(p_{it}^L, p_t^M)/\partial p_t^M}{\partial C_{2t}(p_{it}^L, p_t^M)/\partial p_{it}^L} \omega_{it}^L$$

Note that this function only depends on the input prices and labor productivity, ω_{it}^L. Thus, the flexible input ratio becomes

$$\frac{M_{it}}{L_{it}} = r_t(p_t^L, p_t^M, \omega_{it}^L) = r_t(\omega_{it}^L)$$

where $r_t()$ is an unknown function. As $r_t()$ function is time-varying, and input prices are constant across firms due to the competitive input market, these prices will be a constant in this function. Therefore, the material-to-labor ratio becomes a function of labor augmenting productivity ω_{it}^L.

A.2 Proof of Equation 2.4

I will now show that the material-to-labor ratio is monotonous in ω_{it}^L in order to write labor productivity as a function of the material-to-labor ratio.

Since the cost function is a homogeneous of degree with one, the derivative of the cost function with respect to the input prices would be homogeneous of degree zero. Using this property, dividing by p_t^M will not change the partial derivative of the cost function with respect to flexible inputs. Therefore, I can rewrite it as follows

$$\frac{M_{it}}{L_{it}} = \frac{\partial C_{2t}(\frac{p_{it}^L}{p_t^M}, \frac{p_t^M}{p_t^M})/\partial p_t^M}{\partial C_{2t}(\frac{p_{it}^L}{p_t^M}, \frac{p_t^M}{p_t^M})/\partial p_{it}^L}\omega_{it}^L = \frac{\partial \tilde{C}_{2t}(\frac{p_{it}^L}{p_t^M})/\partial p_t^M}{\partial \tilde{C}_{2t}(\frac{p_{it}^L}{p_t^M})/\partial p_{it}^L}\omega_{it}^L$$

where $\tilde{C}_{2t}(\frac{p_{it}^L}{p_t^M}) = C_{2t}(\frac{p_{it}^L}{p_t^M}, 1)$. For the sake of easy notation, let $\tilde{C}_{2t}(\frac{p_{it}^L}{p_t^M})/\partial p_t^M = \tilde{C}_{2m}$, $\tilde{C}_{2t}(\frac{p_{it}^L}{p_{it}^L})/\partial p_t^M = \tilde{C}_{2l}$, and $\tilde{M}_{it} = \frac{M_{it}}{L_{it}}$. Taking the logarithm of the equation I can rewrite it as

$$log(\tilde{M}_{it}) = log\left(\frac{\tilde{C}_{2m}}{\tilde{C}_{2l}}\right) + log(\omega_{it}^L)$$

Taking the derivative of the equation with respect to $log(\omega_{it}^L)$

$$\frac{\partial log(\tilde{M}_{it})}{\partial log(\omega_{it}^L)} = \frac{\partial \tilde{C}_{2m}/\tilde{C}_{2l}}{\partial log(p_{it}^L/p_t^M)}\frac{log(\partial p_{it}^L/p_t^M)}{\partial log(\omega_{it}^L)} + 1$$

Since $\frac{log(\partial p_{it}^L/p_t^M)}{\partial log(\omega_{it}^L)} = 1$ and $\frac{\partial \tilde{C}_{2m}/\tilde{C}_{2l}}{\partial log(p_{it}^L/p_t^M)} = -\sigma(\omega_{it}^L L_{it}, M_{it})$ is the elasticity of substitution between effective labor and material, the equation becomes

$$\frac{\partial log(\tilde{M}_{it})}{\partial log(\omega_{it}^L)} = -\sigma(\omega_{it}^L L_{it}, M_{it}) + 1$$

From the elasticity substitution assumption $\sigma(\omega_{it}^L L_{it}, M_{it}) > 1$ or $\sigma(\omega_{it}^L L_{it}, M_{it}) < 1$, the derivative of $\tilde{M}$ with respect to ω_{it}^L is either positive or negative. IT will never become zero. This implies that $\tilde{M}_{it}$ is strictly increasing or decreasing function of ω_{it}^L. Thus, $\tilde{M}_{it}$ is monotone in ω_{it}^L. Then, we can invert the $r_t(\omega_{it}^L)$ function in ω_{it}^L, then labor productivity, ω_{it}^L becomes

$$\omega_{it}^L = r_t^{-1}\left(\frac{M_{it}}{L_{it}}\right) = r_t^{-1}(\tilde{M}_{it})$$

Thus, this shows that labor productivity is a function of the material-to-labor ratio.

A.3 Proof of Equation 4.1

The cost minimization problem of the firm i at time t is

$$\min_{L_{it}, M_{it}} \left\{ p_t^L L_{it} + p_t^M M_{it} : F_t(K_{it}, K_{it}^{int}, h_t(\omega_{it}^L L_{it}, M_{it})) exp(\omega_{it}^H)\mathbb{E}[exp(\epsilon_{it})\mathcal{I}_{it}] \geq \bar{Y}_{it} \right\}$$

The first order condition for material and labor is

$$\begin{aligned} p_t^M &= \lambda_{it}\frac{\partial F_t(K_{it}, K_{it}^{int}, h_t(\omega_{it}^L L_{it}, M_{it})) exp(\omega_{it}^H)\mathbb{E}[exp(\epsilon_{it})\mathcal{I}_{it}]}{\partial M_{it}} \\ p_t^L &= \lambda_{it}\frac{\partial F_t(K_{it}, K_{it}^{int}, h_t(\omega_{it}^L L_{it}, M_{it})) exp(\omega_{it}^H)\mathbb{E}[exp(\epsilon_{it})\mathcal{I}_{it}]}{\partial L_{it}} \end{aligned}$$

where λ_{it} is the Lagrange multiplier on the constraint. λ_{it} will capture the firms' marginal cost. Multiplying the first equation by $\frac{M_{it}}{p_{it}Y_{it}}$ and the second equation by $\frac{M_{it}}{p_{it}Y_{it}}$

$$\begin{aligned} \frac{p_t^M M_{it}}{p_{it}Y_{it}} &= \lambda_{it}\frac{\partial F_t(K_{it}, K_{it}^{int}, h_t(\omega_{it}^L L_{it}, M_{it})) exp(\omega_{it}^H)\mathbb{E}[exp(\epsilon_{it})\mathcal{I}_{it}]}{\partial M_{it}}\frac{M_{it}}{p_{it}Y_{it}} \\ \frac{p_t^L L_{it}}{p_{it}Y_{it}} &= \lambda_{it}\frac{\partial F_t(K_{it}, K_{it}^{int}, h_t(\omega_{it}^L L_{it}, M_{it})) exp(\omega_{it}^H)\mathbb{E}[exp(\epsilon_{it})\mathcal{I}_{it}]}{\partial L_{it}}\frac{L_{it}}{p_{it}Y_{it}} \end{aligned}$$

The right-hand side of the first equation becomes the material share in revenue. Similarly, the right-hand side of the second equation becomes labor share in revenue. Let's denote them as α_{it}^M and α_{it}^L as the material and labor share in revenue, respectively. On the left-hand side of the equations, $\frac{\partial F_t()}{\partial M_{it}}\frac{M_{it}}{Y_{it}}$ is the output elasticity of material, and similarly $\frac{\partial F_t()}{\partial L_{it}}\frac{L_{it}}{Y_{it}}$ is the output elasticity of labor. Let's denote ϵ_{it}^L, and ϵ_{it}^M as the labor and material elasticity, respectively. Thus, we have

$$\begin{aligned} \alpha_{it}^M &= \frac{\lambda_{it}}{p_{it}}\epsilon_{it}^M \\ \alpha_{it}^L &= \frac{\lambda_{it}}{p_{it}}\epsilon_{it}^L \end{aligned} \tag{A.3}$$

Taking their ratios we will have

$$\frac{\alpha_{it}^M}{\alpha_{it}^L} = \frac{\epsilon_{it}^M}{\epsilon_{it}^L} \tag{A.4}$$

This shows that the ratio of revenue shares of flexible inputs is equal to their ratio of output elasticities.

A.4 Proof of Equation 4.3

In the equation A.5, $\frac{\lambda_{it}}{p_{it}}$ is equal to the inverse markup because the markup is defined as price over marginal cost. Since λ_{it} is the marginal cost of the firm markup becomes $\mu_{it} = \frac{p_{it}}{\lambda_{it}}$ by definition. Therefore, we can rewrite equation A.5 as

$$\alpha_{it}^M = \frac{\epsilon_{it}^M}{\mu_{it}}$$
$$\alpha_{it}^L = \frac{\epsilon_{it}^L}{\mu_{it}} \tag{A.5}$$

Rearranging the equations markups can be rewritten as

$$\mu_{it} = \frac{\epsilon_{it}^M}{\alpha_{it}^M} = \frac{\epsilon_{it}^L}{\alpha_{it}^L}$$

The markup charged by the firms equals the ratio of their flexible input elasticity to the flexible input's share in revenue.

A.5 Proof of Proposition 4.1

For the production function,

$$y_{it} = f_t(K_{it}, K_{it}^{int}, L_{it}h_t(\omega_{it}^L, \tilde{M}_{it})) + \omega_{it}^H + \epsilon_{it}$$

The material and labor elasticity are

$$\theta_{it}^M = f_{t3}h_{t2}M_{it}$$
$$\theta_{it}^L = f_{t3}(h_t - h_{t2}\tilde{M}_{it})L_{it}$$

where f_{t3} is the derivative of f_t function with respect to the third arguments and h_{t2} is the derivative of h_t function with respect to the second argument. The sum of labor and material elasticity is

$$\theta_{it}^M + \theta_{it}^L = f_{t3} h_{t2} M_{it} + f_{t3}(h_t - h_{t2}\tilde{M}_{it}) L_{it} = f_{t3} h_t L_{it} = f_{t3} \bar{h}_t L_{it}$$

Since the sum of elasticities do not depend on the derivative of h_t function I can replace $h_t()$ with $\bar{h}_t()$. The last equality comes from that fact.

B Data and Estimation

B.1 Estimation Strategy

This section provides a detailed estimation strategy employed. I first construct the control variables, u_{it}^1 and u_{it}^2 to control for Hicks-neutral and labor augmenting productivity. u_{it}^1 is defined in equation 3.1 as

$$u_{it}^1 = F_{\tilde{M}_{it}|W_{it-1}, \sum_{j\in I, j\neq i} R_{jt-1}} \left(\tilde{M}_{it} \mid W_{it-1}, \sum_{j\in I, j\neq i} R_{jt-1} \right)$$

u_{it}^1 is computed as a conditional CDF of $\tilde{M}$ given $W_{it-1}, \sum_{j\in I, j\neq i} R_{jt-1}$. However, the dependent variable, $\tilde{M}_{it}$ is a continuous variable so I can't directly apply the logistic regression. To do that, I first split the data into year and sector subsets since I do production function estimation at the sector year level. I then partition the sub-sample data (within a year and sector) into 500 parts. In cases where 500 partitioning is not feasible I partition them into the one tenth of the sample size[2]. At the boundary points of each discretized data points (named as $q \in Q$) I construct the logistic regression as follow

$$P(\tilde{M}_{it} \leq q \mid W_{it-1} = w, \sum_{j\in I, j\neq i} R_{jt-1} = r) = s_{1t}(q, w, r)$$

The dependent variable becomes a discrete variable depending on whether $\tilde{M}_{it}$ is less than q. I approximate the s_{2t} function with the second-order polynomials[3]. I, then, estimate the predicted CDF for each q points using logistic regression. I compute the remaining points using a linear interpolation.

[2] I use different partition size such as 5. They don't change the results

[3] Third-order approximation gives the same results.

To estimate the u_{it}^2 I follow the similar strategy as u_{it}^1. The equation 3.3 defines u_{it}^2 as follow

$$u_{it}^2 = F_{M_{it}|K_{it},K_{it}^{int},W_{it-1},\sum_{j\in I,j\neq i} R_{jt-1}}\Big(M_{it} \mid K_{it}, K_{it}^{int}, W_{it-1}, \sum_{j\in I,j\neq i} R_{jt-1}\Big)$$

As in u_{it}^1, the dependent variable M_{it} is not a discrete variable, it is a continuous variable. I partition the data 500 parts at the sector year level. If the data is not feasible I split it into the one-tenth of the data. At each q level, I do

$$P(M_{it} \leq q \mid K_{it} = k, K_{it}^{int} = k^{int}, W_{it-1} = w, \sum_{j\in I,j\neq i} R_{jt-1} = r, u_{it}^1 = u^1) = s_{2t}(q, k, k^{int}, w, r, u^1)$$

The dependent variable becomes a discrete variable depending on whether M_{it} is less than q. I similarly approximate s_{2t} function with second order polynomials. I then find the CDF at the points where $M_{it} = q$. For the remaining points, I compute the linear interpolation so that I have an estimator for u_{it}^2 at each point in the data.

After estimating u_{it}^1 and u_{it}^2, I can now estimate the productions functions. I estimate the productions at each sector over 7-year rolling windows in the equation 5.4

$$y_{it} = f_t(K_{it}, K_{it}^{int}, L_{it}\bar{h}_t(\tilde{M}_{it})) + c_{2t}\big(W_{it-1}, \sum_{j\in I,j\neq i} R_{jt-1}, u_{it}^1, u_{it}^2\big) + \epsilon_{it}$$

If I know $\bar{h}_t(.)$ function, I can estimate the regression using polynomial approximations for $f_t()$ and $c_{2t}()$ functions. I first approximate the $h_t()$ function with third-order polynomials.

$$h_t(\tilde{m}_{it}) = \alpha_{1t}\tilde{m}_{it}^2 + \alpha_{3t}\tilde{m}_{it}^3$$

I approximate the $f_t()$ function with second-order polynomials. For the sake of easy notation, let's define $v_{it} = l_{it} + h(m_{it})$. With the approximated $h_t()$ given, I can write $f_t()$ as follow

$$\begin{aligned} f_t() = \beta_{0t} + \beta_{1t}k_{it} &+ \beta_{2t}k_{it}^{int} + \beta_{3t}v_{it} + \beta_{4t}k_{it}^2 + \beta_{5t}k_{it}k_{it}^{int} + \beta_{6t}k_{it}v_{it} \\ &+ \beta_{7t}(k_{it}^{int})^2 + \beta_{8t}k_{it}^{int}v_{it} + \beta_{9t}v_{it}^2 \end{aligned}$$

where β's represent the coefficients of the polynomial approximations. I have time subscripts because I estimate them over 7 year rolling windows within each sector. These coefficients will be sector-specific and time varying. I similarly approximate c_{2t} and c_{3t} with second-order polynomials. I do then minimize the moment conditions with these approximated functions in equation 5.7.

I estimate the production function parameters β's and $c_2()$ and $c_3()$ coefficients given α's. I first guess α values and then estimate the moment conditions using the least squares. I iterate the loops to minimize the moment conditions in equation 5.7. This will give me the β coefficients and productivity function coefficients. This is not a very hard loop because given $alphas$ it is straightforward to find other parameters. Therefore, in the loops we are minimizing the OLS estimates to find 3 α parameters to minimize the residuals.

After estimating the parameters of $f_2()$ and c_2 functions, I can compute the elasticities as defined in the identification section. The output elasticity of intangible and tangible capital is the derivative of estimated f_t function with respect to the capital and intangible capital, respectively. Since I know that the approximated the f_t function the capital and intangible elasticity are

$$\theta_{it}^{k} = \beta_{1t} + 2\beta_{4t}k_{it} + \beta_{5t}k_{it}^{int} + \beta_{6t}v_{it}$$
$$\theta_{it}^{int} = \beta_{2t} + \beta_{5t}k_{it}^{int} + 2\beta_{7t}k_{it}^{int} + \beta_{8t}v_{it}$$

Due to the input heterogeneity this generates elasticity heterogeneity across firms. For the identification of flexible inputs I similarly take the derivative of f_t function with respect to v, this would give me the sum of flexible input elasticities. Multiplying with the flexible share in revenue I compute the labor and material elasticities. The productivity elasticity of the external knowledge is the derivative of c_2 function with respect to external knowledge capital. I construct them similar to the output elasticities.

B.2 Data

I use the Compustat data from Standard and Poor's Compustat North America database from 1975 to 2020. The Compustat data covers all public firms in the US. The data is available before 1975, but since intangible asset's parameters are estimated using a data from 1975 in Ewens et al. (2019), I exclude the firms before 1975. I clean the data as standard in the literature. I drop the firms that are not operating in the US firms. I drop financial and utility companies with SIC codes between 6000-7000, and 4900-5000. I remove the firms with zero or negative sales, cogs, ppegt, intangible capital, xsga and employment. I further exclude too small firms having employees less than 10. I, then, drop the firms that do not have naics industry classification codes. I remove the firms at the bottom 1% of the variables to omit the outliers. I do not exclude top 1% because some of the giant firms such as Amazon, Apple exclude from the data. Therefore, I only trim the data at the bottom 1%.

In the production function estimation I need output, physical capital, intangible capital, labor, material and knowledge capital. I use sale(Sales/Turnover (Net)) variable in Compustat for output. For the physical capital I use ppegt(Property, Plant and Equipment - Total (Gross)) variable. I use cogs(cost of goods sold) variable for the material. emp (Employees in the Compustat captures the labor in the production function. In the data section I explain how to construct the intangible capital. Knowledge capital is the capital stock constructed xrd (Research & Development Expense variables in Compustat. I deflate all the variables with corresponding industry specific deflators in the BEA. Table A1 represents some summary statistics for the variables used in my estimations(sales, ppegt, intantible capital, emp) after deflating and cleaning the data. Table A2 respresents the intangible ratio summary statistics. On average intangible capital to tangible capital ratio is 0.44 showing the importance of intangible capital in the firm's capital stocks. Table A3 shows the median firm's intangible ratio, firm's assets, age and investment rate and employment level.

Table A1: Summary Statistics - Compustat Variables

	Mean	P25	P50	P75	Count
Assets - Total (million $)	2701.215	30.68	129.815	736.701	225924
Market Value (million $)	4516.595	45.864	195.586	1152.329	194817
Sales/Turnover (Net) (million $)	2173.114	29.588	132.168	709.678	225924
Employees (thousands)	10.181	.257	1.15	5.177	211522
Property, Plant and Equipment - Total (Net) (million $)	938.432	5.122	27.214	198.112	225526
Capital Expenditures (million $)	165.572	1.075	5.908	37.727	223374
Intangible Capital (million $)	593.076	6.057	27.466	137.155	225924
Research and Development Expense (million $)	51.925	0	0	5.163	225924
Selling, General and Administrative Expense (million $)	289.917	4.703	19.64	96.428	225924
Other Intangibles (million $)	169.731	0	0	.045	225924
Cash per Assets - Total	.164	.026	.078	.215	225804
Leverage per Assets - Total	.271	.062	.225	.389	225122
Tobin's Q	1.092	.184	.62	1.287	195043
Dividends per Assets - Total	.012	0	0	.012	225924
Repurchases per Assets - Total	-.043	-.008	0	0	206447
Total Payouts per Assets - Total	-.03	-.005	0	.018	206447
Retained Earnings per Assets - Total	-.387	-.178	.132	.339	221681

Note: This table documents the summary statistics of some selected firm-level variables in the Compustat. P25: 25^{th} percentile, P50: median and P75: 75^{th} percentile.

Table A2: Summary Statistics - Intangible Capital Ratio

	Mean	Sd	P25	P50	P75	Min	Max	Count
Intangible Ratio	.446	.292	.184	.486	.7	0	1	202315

Note: This table documents the summary statistics of intangible ratio. p25: 25^{th} percentile, p50: median and p75: 75^{th} percentile.

Table A3: Summary Statistics by Intangible Capital Ratio Quintiles

Quintiles	Intangible Ratio	Total Asset	Age	Total Investment Rate	Employment
Q1	0	702	22	.14	1.5
Q2	.22	272	19	.23	1.7
Q3	.5	273	20	.29	1.5
Q4	.72	145	19	.33	1.1
Q5	.91	41	16	.34	.25
Total	.49	185	19	.27	1

Note: This table documents the pool sample median of some selected firm-level variables within each quintile of intangible capital ratio. Q1 is the bottom quintile and Q5 is the top quintile in terms of intangible capital ratio. Intangible ratio is defined as $\frac{\text{Intangible capital stock}}{\text{Intangible capital stock} + \text{Tangible capital stock}}$ where intangible capital stock is constructed based on the perpetual inventory method of Peters and Taylor (2017b). Tangible capital stock is the total net plant,property and equipment.

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